OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4768

Statistics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Tuesday 22 June 2010
Afternoon

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(i)** The manager of a company that employs 250 travelling sales representatives wishes to carry out a detailed analysis of the expenses claimed by the representatives. He has an alphabetical (by surname) list of the representatives. He chooses a sample of representatives by selecting the 10th, 20th, 30th and so on. Name the type of sampling the manager is attempting to use. Describe a weakness in his method of using it, and explain how he might overcome this weakness. **[3]**

The representatives each use their own cars to drive to meetings with customers. The total distance, in miles, travelled by a representative in a month is Normally distributed with mean 2018 and standard deviation 96.

(ii) Find the probability that, in a randomly chosen month, a randomly chosen representative travels more than 2100 miles. **[3]**

(iii) Find the probability that, in a randomly chosen 3-month period, a randomly chosen representative travels less than 6000 miles. What assumption is needed here? Give a reason why it may not be realistic. **[5]**

(iv) Each month every representative submits a claim for travelling expenses plus commission. Travelling expenses are paid at the rate of 45 pence per mile. The commission is 10% of the value of sales in that month. The value, in £, of the monthly sales has the distribution $N(21\,200,\ 1100^2)$. Find the probability that a randomly chosen claim lies between £3000 and £3300. **[7]**

2 William Sealy, a biochemistry student, is doing work experience at a brewery. One of his tasks is to monitor the specific gravity of the brewing mixture during the brewing process. For one particular recipe, an initial specific gravity of 1.040 is required. A random sample of 9 measurements of the specific gravity at the start of the process gave the following results.

1.046 1.048 1.039 1.055 1.038 1.054 1.038 1.051 1.038

(i) William has to test whether the specific gravity of the mixture meets the requirement. Why might a t test be used for these data and what assumption must be made? **[3]**

(ii) Carry out the test using a significance level of 10%. **[9]**

(iii) Find a 95% confidence interval for the true mean specific gravity of the mixture and explain what is meant by a 95% confidence interval. **[6]**

3 **(a)** In order to prevent and/or control the spread of infectious diseases, the Government has various vaccination programmes. One such programme requires people to receive a booster injection at the age of 18. It is felt that the proportion of people receiving this booster could be increased and a publicity campaign is undertaken for this purpose. In order to assess the effectiveness of this campaign, health authorities across the country are asked to report the percentage of 18-year-olds receiving the booster before and after the campaign. The results for a randomly chosen sample of 9 authorities are as follows.

Authority	A	B	C	D	E	F	G	H	I
Before	76	98	88	81	86	84	83	93	80
After	82	97	93	77	83	95	91	95	89

This sample is to be tested to see whether the campaign appears to have been successful in raising the percentage receiving the booster.

(i) Explain why the use of paired data is appropriate in this context. **[1]**

(ii) Carry out an appropriate Wilcoxon signed rank test using these data, at the 5% significance level. **[10]**

(b) Benford's Law predicts the following probability distribution for the first significant digit in some large data sets.

Digit	1	2	3	4	5	6	7	8	9
Probability	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

On one particular day, the first significant digits of the stock market prices of the shares of a random sample of 200 companies gave the following results.

Digit	1	2	3	4	5	6	7	8	9
Frequency	55	34	27	16	15	17	12	15	9

Test at the 10% level of significance whether Benford's Law provides a reasonable model in the context of share prices. **[7]**

[Question 4 is printed overleaf.]

4 A random variable X has an exponential distribution with probability density function $f(x) = \lambda e^{-\lambda x}$ for $x \geqslant 0$, where λ is a positive constant.

(i) Verify that $\int_0^{\infty} f(x)\,dx = 1$ and sketch $f(x)$. **[5]**

(ii) In this part of the question you may use the following result.

$$\int_0^{\infty} x^r e^{-\lambda x}\,dx = \frac{r!}{\lambda^{r+1}} \quad \text{for } r = 0,\ 1,\ 2,\ \ldots\ .$$

Derive the mean and variance of X in terms of λ. **[6]**

The random variable X is used to model the lifetime, in years, of a particular type of domestic appliance. The manufacturer of the appliance states that, based on past experience, the mean lifetime is 6 years.

(iii) Let $\overline{X}$ denote the mean lifetime, in years, of a random sample of 50 appliances. Write down an approximate distribution for $\overline{X}$. **[4]**

(iv) A random sample of 50 appliances is found to have a mean lifetime of 7.8 years. Does this cast any doubt on the model? **[3]**

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **4768**

Statistics 3

Mark Scheme for June 2010

Q1	D ~ N(2018, σ = 96)		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	Systematic Sampling. It lacks any element of randomness. Choose a random starting point in the range 1 – 10.	B1 E1 E1	 May be implied by the next mark. Allow reasonable alternatives e.g. "the list may contain cycles." Beware proposals for a different sampling method.	[3]
(ii)	P(D > 2100) = $\text{P}\left(Z > \frac{2100 - 2018}{96} = 0.8542\right)$ = 1 − 0.8034 = 0.1966	M1 A1 A1	For standardising. Award once, here or elsewhere. c.a.o.	[3]
(iii)	$D_1 + D_2 + D_3 \sim \text{N}(6054,$ $\sigma^2 = 96^2 + 96^2 + 96^2 = 27648$) P(this < 6000) = $\text{P}\left(Z < \frac{6000 - 6054}{166.277} = -0.3248\right)$ = 1 − 0.6273 = 0.3727 Must assume that the months are independent. This is unlikely to be realistic since e.g. consecutive months may not be independent.	B1 B1 A1 E1 E1	Mean. Variance. Accept sd (= 166.277). c.a.o. Reference to independence of months. Any sensible comment.	[5]
(iv)	Claim ~ N(2018 × 0.45 + 21200 × 0.10 = 3028.10, $96^2 \times 0.45^2 + 1100^2 \times 0.10^2 = 13966.24$ P(3000 < this < 3300) $= \text{P}\left(\frac{3000 - 3028.1}{118.18} < Z < \frac{3300 - 3028.1}{118.18}\right)$ $= \text{P}(-0 \cdot 2378 < Z < 2.3008)$ = 0.9893 − (1 − 0.5940) = 0.5833	M1 A1 M1 A1 M1 A1 A1	Mean. c.a.o. Variance. Accept sd (= 118.18). c.a.o. Formulation of requirement: a two-sided inequality. Ft c's parameters. c.a.o.	[7]
			Total	**[18]**

Q2				
(i)	A t test might be used because • sample is small • population variance is unknown Must assume background population is Normal.	 B1 B1 B1		[3]
(ii)	H_0: $\mu = 1.040$ H_1: $\mu \neq 1.040$	B1	Both hypotheses. Hypotheses in words only must include "population". Do NOT allow "$\overline{X} = ...$" or similar unless $\overline{X}$ is clearly and explicitly stated to be a population mean.	
	where μ is the mean specific gravity of the mixture.	B1	For adequate verbal definition. Allow absence of "population" if correct notation μ is used.	
	$\bar{x} = 1.0452$ $s_{n-1} = 0.007155$	B1	$s_n = 0.006746$ but do NOT allow this here or in construction of test statistic, but FT from there.	
	Test statistic is $\dfrac{1.0452 - 1.040}{\frac{0.007155}{\sqrt{9}}}$	M1	Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 1.040 + (c's 1.860) × $\frac{0.007155}{\sqrt{9}}$ (= 1.0444) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's 860) × $\frac{0.007155}{\sqrt{9}}$ (= 1.0407) for comparison with 1.040.)	
	= 2.189(60).	A1	c.a.o. but ft from here in any case if wrong. Use of 1.040 – $\bar{x}$ scores M1A0, but ft.	
	Refer to t_8.	M1	No ft from here if wrong. $P(t > 2.1896) = 0.05996$.	
	Double-tailed 10% point is 1.860.	A1	No ft from here if wrong.	
	Significant.	A1	ft only c's test statistic.	
	Seems mean specific gravity in the mixture does not meet the requirement.	A1	ft only c's test statistic.	[9]
(iii)	CI is given by $1.0452 \pm$	M1		
	$2{\cdot}306$	B1		
	$\times \dfrac{0.007155}{\sqrt{9}}$	M1		
	$= 1.0452 \pm 0.0055 = (1.039(7), 1.050(7))$	A1	c.a.o. Must be expressed as an interval. ZERO/4 if not same distribution as test. Same wrong distribution scores maximum M1B0M1A0. Recovery to t_8 is OK.	
	In repeated sampling, 95% of confidence intervals constructed in this way will contain the true population mean.	E2	E2, 1, 0.	[6]
			Total	**[18]**

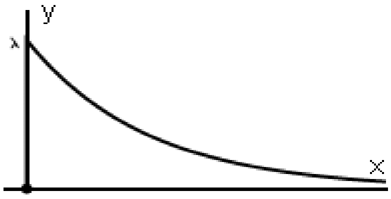
Q3			
(a) (i)	Use paired data in order to eliminate differences between authorities.	B1	[1]
(ii)	H_0: $m = 0$ H_1: $m > 0$ where m is the population median difference.	B1 Both. Accept hypotheses in words. B1 Adequate definition of m to include "population".	

Diff (After − Before)	6	−1	5	−4	−3	11	8	2	9
Rank of |diff|	6	1	5	4	3	9	7	2	8

		M1 For differences. ZERO in this section if differences not used. M1 For ranks. A1 FT from here if ranks wrong	
	$W_- = 1 + 3 + 4 = 8$ (or = 2+5+6+7+8+9 = 37)	B1	
	Refer to tables of Wilcoxon paired (/single sample) statistic for $n = 9$.	M1 No ft from here if wrong.	
	Lower 5% point is 8 (or upper is 37 if W_+ used).	A1 i.e. a 1-tail test. No ft from here if wrong.	
	Result is significant.	A1 ft only c's test statistic.	
	Evidence suggests the percentage has been raised (on the whole).	A1 ft only c's test statistic.	[10]
(b)	H_0: Stock market prices can be modelled by Benford's Law. H_1: Stock market prices can not be modelled by Benford's Law.		

Prob	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046
Exp f	60.2	35.2	25.0	19.4	15.8	13.4	11.6	10.2	9.2
Obs f	55	34	27	16	15	17	12	15	9

		M1 Probs × 200 for expected frequencies. All correct.	
	$X^2 = 0.44917 + 0.04091 + 0.16 + 0.59588 + 0.04051 + 0.96716 + 0.01379 + 2.25882 + 0.00435 = 4.5305(9)$	M1 Calculation of X^2. A1 c.a.o.	
	Refer to χ^2_8.	M1 Allow correct df (= cells – 1) from wrongly grouped table and ft. Otherwise, no ft if wrong. $P(X^2 > 4.53059) = 0.80636$.	
	Upper 5% point is 13.36.	A1 No ft from here if wrong.	
	Not significant.	A1 ft only c's test statistic.	
	Suggests Benford's Law provides a reasonable model in the context of share prices.	A1 ft only c's test statistic.	[7]
		Total	**[18]**

Q4	$f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$, where $\lambda > 0$.	Given $\int_0^\infty x^r e^{-\lambda x} dx = \frac{r!}{\lambda^{r+1}}$	
(i)	$\int_0^\infty f(x)dx = \int_0^\infty \lambda e^{-\lambda x} dx$ $= \left[-e^{-\lambda x}\right]_0^\infty$ $= \left(0 - (-e^0)\right) = 1$ 	M1 Integration of f(x). M1 Use of limits or the given result. A1 Convincingly obtained (Answer given.) G1 Curve, with negative gradient, in the first quadrant only. Must intersect the y-axis. G1 (0, λ) labelled; asymptotic to x-axis.	**[5]**
(ii)	$E(X) = \int_0^\infty \lambda x e^{-\lambda x} dx$ $= \lambda \frac{1}{\lambda^2} = \frac{1}{\lambda}$ $E(X^2) = \int_0^\infty \lambda x^2 e^{-\lambda x} dx$ $= \lambda \frac{2}{\lambda^3} = \frac{2}{\lambda^2}$ $Var(X) = E(X^2) - E(X)^2 = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2}$	M1 Correct integral. A1 c.a.o. (using given result) M1 Correct integral. A1 c.a.o. (using given result) M1 Use of $E(X^2) - E(X)^2$ A1	**[6]**
(iii)	$\mu = 6 \quad \therefore \lambda = \frac{1}{6}$ $\overline{X}$ ~ (approx) $N\left(6, \frac{6^2}{50}\right)$	B1 Obtained λ from the mean. B1 Normal. B1 Mean. ft c's λ. B1 Variance. ft c's λ.	**[4]**
(iv)	**<u>EITHER</u>** can argue that 7.8 is more than 2 SDs from μ. ($6 + 2\sqrt{0.72}$ = 7.697; <u>must</u> refer to SD ($\overline{X}$), not SD(X)) i.e. outlier. ⇒ doubt. **<u>OR</u>** formal significance test: $\frac{7.8 - 6}{\sqrt{0.72}} = 2.121$, refer to N(0,1), sig at (eg) 5% ⇒ doubt.	M1 A 95% C.I would be (6.1369, 9.4631). M1 A1 M1 M1 Depends on first M, but could imply it. P(|Z| > 2.121)= 0.0339 A1	**[3]**
		Total	**[18]**